Harry Potter and the Magic of Mathematics

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THE MIDDLE SCHOOL YEARS REPRESENT an important time in the formation of an individual's lifelong attitudes toward mathematics. Middle school teachers are faced with the challenge of convincing their students that mathematics is an exciting, useful, and creative field of study. Interdisciplinary approaches to mathematics have been useful in accomplishing this goal. In particular, connecting mathematics to literature is an inventive way to capture students' interests, since examples from literature can be used to teach important mathematical concepts in an exciting and innovative manner. Many classic literary texts are rich in mathematical content, including *Alice in Wonderland* by Lewis Carroll and *Flatland* by Edwin A. Abbott. However, to stimulate students' interest, it is important to find interconnections between mathematics and current popular children's literature.

The Harry Potter series by J. K. Rowling is the latest craze in children's literature. The books contain a great deal of creativity and stir a reader's imagination and wonder. The world of Harry Potter is magical, and Harry, himself, is a natural problem solver. Our goal is to use examples from this series to show students the magic of mathematics and to teach them that problem-solving skills are the key to mathematical success.

The popularity of the Harry Potter series makes it possible to use examples from the books in the classroom without assigning the text as additional required reading. In our experiences implementing these exercises, all the middle school students and teachers were familiar with the storyline of the books. The series features an eleven-year-old orphan named Harry Potter, who discovers that his parents had magical powers and that he was born a wizard. The series chronicles Harry's adventures in magic at Hogwarts School of Witchcraft and Wizardry, and it provides an excellent backdrop for many interesting mathematics problems. When presenting our examples to the students, we choose to include many quotations and story scenarios from the Harry Potter series. These references heighten the interest level of our students and make them feel a part of Harry's magical and mathematical world.

In *Harry Potter and the Sorcerer's Stone* (Rowling 1997), the first book in the series, we are introduced to the new world that Harry is experiencing as a student wizard. One of Harry's first challenges is to adjust to the monetary system. Harry is introduced to wizard money when he visits Gringotts, a wizards' bank:

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Griphook unlocked the door. A lot of green smoke came billowing out, and as it cleared, Harry gasped. Inside were mounds of gold coins. Columns of silver. Heaps of little bronze Knuts.

"All yours," smiled Hagrid. . . .

Hagrid helped Harry pile some of it into a bag.

"The gold ones are Galleons," he explained. "Seventeen silver Sickles to a Galleon and twenty-nine Knuts to a Sickle, it's easy enough." (Rowling 1997, p. 75)

One of the first exercises that we do with our students involves exploring the conversions among different kinds of "Potter money." A typical conversion problem would be worded like this:

> A Firebolt broomstick costs 3 gold Galleons, and Harry had only brought silver Sickles and bronze Knuts. How many of these would he need to buy the Firebolt broomstick?

By applying the conversion, they would find a total of 51 silver Sickles (3 Galleons × 17 Sickles per Galleon). After discussing this conversion activity, the class agrees that multiple solutions are possible, for example, 51 silver Sickles; 34 silver Sickles and 493 bronze Knuts; 17 silver Sickles and 986 bronze Knuts; or 1479 bronze Knuts (McShea and Yarnevich 2003, p. 18). (For a detailed look at conversions with "Potter money," see pp. 16–19.)

Middle school students should become proficient in expressing measurements in equivalent forms, such as hours to minutes or feet to inches. By using "Potter money" as an exploratory exercise, students learn these important skills in an interesting and interactive environment. For example, students grasp the equivalence concept of 1 silver Sickle being equal to 29 bronze Knuts in Harry's world, and they are able to extend this concept to conversion problems in our world.

Functions and Linear Modeling

AFTER STUDENTS BECOME FAMILIAR WITH MONetary conversions using wizard money, we introduce an activity that teaches functions and linear modeling. Important goals in the middle grades include developing a student's ability to recognize patterns, which can be represented by linear functions, and analyzing a variety of real-world relationships using these mathematical models. "With computers and graphing calculators . . . students can focus on using functions to model patterns of quantitative change" (NCTM 2000, p. 227). Examples of functional modeling can be found both in Harry's magical world and in students' real-world experiences in the areas of athletics, finance, and conversions involving money and weather. By introducing functions with the following example from *Harry Potter and the Sorcerer's Stone,* our students gain an appreciation for the creativity involved in finding functional patterns.

On the train ride to Hogwarts School of Witchcraft and Wizardry, Harry experiences his first sense of freedom. Harry's parents died when he was an infant, and he was raised by his Aunt Petunia, Uncle Vernon, and Cousin Dudley Dursley. The Dursleys are not like Harry. They are *Muggles* (humans without one drop of magic in them), and they loathe the idea of taking care of Harry. Harry was forced to live in a cupboard in the Dursleys' house and was given only a limited amount of food. After he was free from the Dursleys and had money in his pocket, Harry had his first opportunity to indulge himself:

> He had never had any money for candy with the Dursleys, and now that he had pockets rattling with gold and silver he was ready to buy as many Mars Bars as he could carry—but the woman didn't have Mars Bars. What she did have were Bertie Bott's Every Flavor Beans, Drooble's Best Blowing Gum, Chocolate Frogs, Pumpkin Pasties, Cauldron Cakes, Licorice Wands, and a number of strange things Harry had never seen in his life. (Rowling 1997, p. 101)

In our exercise, we ask students to help Harry solve the following problem:

> Assume that Harry bought Chocolate Frogs, which cost 11 bronze Knuts per bag, and Bertie Bott's Every Flavor Beans, which cost 17 bronze Knuts per bag. How many bags of each candy did Harry buy if his purchase totaled 11 silver Sickles and 7 bronze Knuts?

Most students begin this problem using the skills learned in the previous conversion exercise. They convert Harry's total purchase of 11 silver Sickles and 7 bronze Knuts into the same "Potter money" unit, or bronze Knuts. Thus, they convert the 11 silver Sickles into 319 bronze Knuts and add the remaining 7 bronze Knuts to get a total purchase price of 326 bronze Knuts. Many groups then attack this problem using a guess-and-check strategy. This technique usually leads them to one of the following possible solutions: 8 bags of Chocolate Frogs and 14 bags of Bertie Bott's Every Flavor Beans or 25 bags of Chocolate Frogs and 3 bags of Bertie Bott's Every Flavor Beans. We find our students will take the 326 bronze Knuts and divide by 11 bronze Knuts, the cost of a bag of Chocolate Frogs ($326 \div 11 = 29.64$ bags of Chocolate Frogs), or divide by 17 bronze Knuts, the cost of a bag of Bertie Bott's Every Flavor Beans ($326 \div 17 = 19.18$ bags of Bertie Bott's Every

TABLE 1
Conversions of Potter Food and Money

No. of Chocolate Frogs	No. of Bertie Bott's Every Flavor Beans	Total Purchase Price in Bronze Knuts	
29	0	319 (29 • 11)	"7 short"
28	1	325 (28 • 11 + 1 • 17)	"1 short"
27	2	331 (27 • 11 + 2 • 17)	"5 too much"
26	2	320 (26 • 11 + 2 • 17)	"6 short"
25	3	326 (25 • 11 + 3 • 17)	**Correct answer**

TABLE 2
More Conversions of Potter Food and Money

No. of Bertie Bott's Every Flavor Beans	No. of Chocolate Frogs	Total Purchase Price in Bronze Knuts	
19	0	323 (19 • 17)	"3 short"
18	1	317 (18 • 17 + 1 • 11)	"9 short"
17	3	322 (17 • 17 + 3 • 11)	"4 short"
16	4	316 (16 • 17 + 4 • 11)	"10 short"
15	6	321 (15 • 17 + 6 • 11)	"5 short"
14	8	326 (14 • 17 + 8 • 11)	**Correct answer**

Flavor Beans). Realizing that neither quantity divides evenly into the total of 326 bronze Knuts, our students will continue to reduce the number of bags bought and add a bag or two of the other candy to try and make up the difference. **Tables 1** and **2** are formal representations of our students' written and oral work. This guess-and-check strategy works in finding the correct solution; however, it is not very efficient. We like to introduce the concept of linear modeling at this point and assign variables for the unknown values.

Principles and Standards for School Mathematics states that "Teachers can help students move from a limited understanding of variable as a placeholder for a single number to the idea of variable as a representation for a range of possible values by providing experiences that use variable expressions to describe numerical data" (Demana and Leitzel 1988). This problem provides such an opportunity. Because it has multiple solutions, we show the students how this problem can be illustrated as a linear equation, using x as the number of bags of Chocolate Frogs purchased and y as the number of bags of Bertie Bott's Every Flavor Beans. We help our students think about the problem by writing the necessary linear equation in words:

cost per bag • number of Chocolate Frogs bought
+ cost per bag • number of Bertie Bott's Every Flavor Beans
= total cost of Harry's purchase

The linear equation with the appropriate variables is $11x + 17y = 326$, or $y = (-11/17)x + 326/17$. By setting up this linear model and using a graphing calculator, the students construct the table of solutions for this problem shown in **figure 1,** which are formal representations of our students' work. From these examples, they can see that all these solutions satisfy the given equation but that not all solutions make sense. Students recognize that decimal values and negative values would not make sense in terms of the number of bags. Therefore, the only solutions that work in the real world are whole-number answers.

We also like to reinforce this activity by discussing the relevance of graphing this problem's linear equation, $y = (-11/17)x + 326/17$ (see **figs. 2a** and **2b**). In a linear equation, where the variables x and y could represent anything, the line displays all possible values for x and y. However, since we are relating the linear equation to the Harry Potter problem, the only values that make

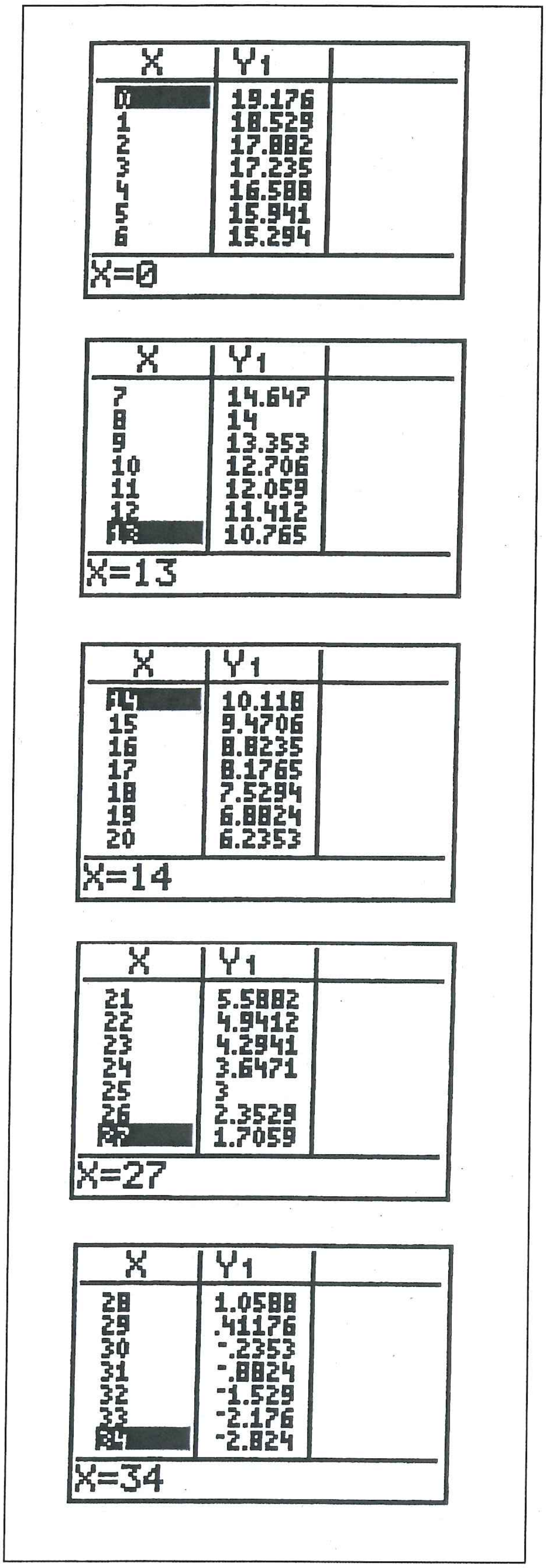

Fig. 1 Students construct a table of solutions for the problem.

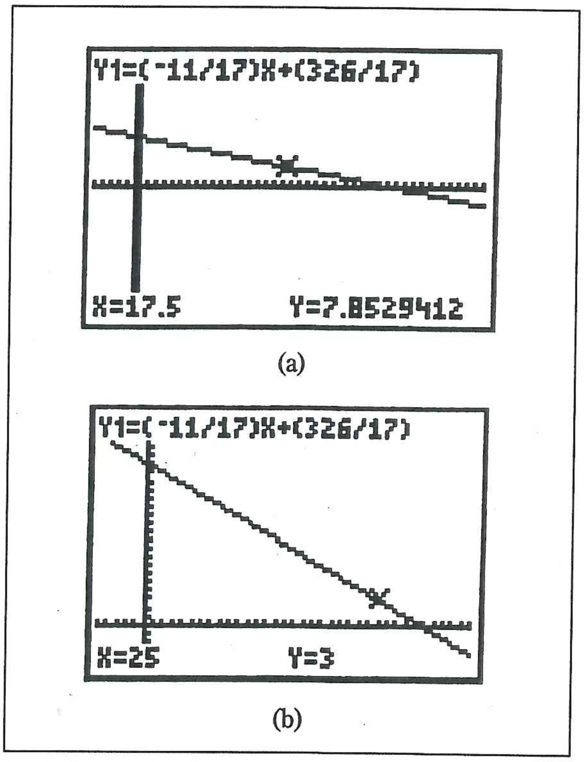

Fig. 2 The graphed linear equation

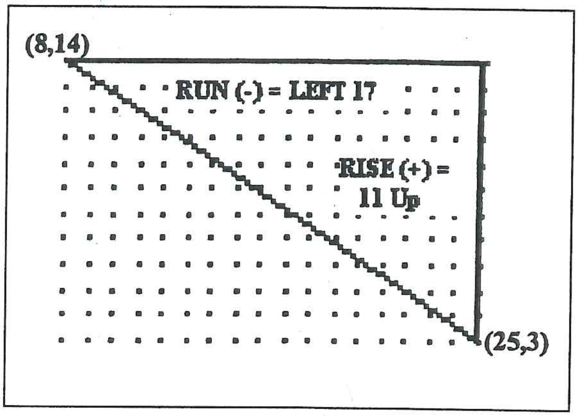

Fig. 3 The rise and run are illustrated.

sense are whole-number solutions, e.g., (25, 3) or $x = 25$ and $y = 3$ (see **fig. 2b**). The point (25, 3) and the slope of the line, –11/17, can be used in relation to the graph as a tool for finding all possible integer solutions. By starting at the point (25, 3) and counting up 11 and left 17, another realistic solution can be found, which is that $x = 8$ and $y = 14$, or (8, 14). This calculation is demonstrated in **figure 3.** In a similar manner, some students may choose to count down 11 and right 17, finding the solution (42, –8). Although (42, –8) is an integer solution to the problem, it does not make sense in the context of the storyline. Therefore, the only values found graphically that make sense are $x = 25$ and $y = 3$, or $x = 8$ and $y = 14$.

Probability

UNDERSTANDING AND CALCULATING PROBABILITIES are key elements in middle school mathematics. Students should be able to recognize situations involving the probability of outcomes and to use a variety of techniques to better understand the probabilities involved. We want our students to investigate patterns in outcomes by performing simple, repeatable experiments, such as flipping a coin, and to extend the patterns into a mathematical formula. We use the following example from the Harry Potter series to emphasize the importance of understanding probabilities.

When new students arrive at the Hogwarts School, they are placed into one of the four houses: Gryffindor, Hufflepuff, Ravenclaw, or Slytherin. A student's house is decided during the Sorting Hat ceremony. "'Move along now,' said a sharp voice. 'The Sorting Ceremony's about to start'" (Rowling 1997, p. 116). The sorting ceremony is a fun-filled event in which a talking hat determines the fate of the students. The ceremony is a magical event for new students, and there is much wonderment when the hat begins to sing:

> "Oh, you may not think I'm pretty,
> But don't judge on what you see,
> I'll eat myself if you can find
> A smarter hat than me. . . .
> For I'm the Hogwarts Sorting Hat . . .
> You might belong in Gryffindor . . .
> You might belong in Hufflepuff . . .
> Or yet in wise old Ravenclaw, . . .
> Or perhaps in Slytherin. . . ."
>
> The whole hall burst into applause as the hat finishes its song. . . . (Rowling 1997, pp. 117–18)

One by one, the hat is placed on each new student's head and calls out the name of a house. After the Sorting Hat ceremony is completed, the main characters in the book—Harry, Ron, and Hermione—are all placed in Gryffindor and become close friends. The plot of the series depends on the trio being placed into Gryffindor. By belonging to the same house, they are able to meet and share in one another's adventures. In addition, one of the main characteristics of the Gryffindor house is bravery, which is the storyline's central theme. After reading the books, our students understand that it is important for these three characters to be together in the Gryffindor house. We then pose the following question:

> Can you figure out the chances that Ron, Harry, and Hermione will all be placed into Gryffindor?

We begin this activity by assigning different students the roles of Ron, Harry, and Hermione. These students then randomly select from a hat one of the four house cards. After making their selections, we list that outcome on the board. We continue this process, having different groups of three come to select their individual house fate. In most cases, the groups choose completely different outcomes. However, at times a group repeats the outcome of a previous group. We emphasize the importance of looking at unique possibilities and have that group pick again. After several attempts, we break our students into groups to work together to find all the possible combinations of how Ron, Harry, and Hermione could be placed into the four different houses.

As the groups work at getting all possible combinations, we notice two popular approaches. Many groups start by listing the possibilities either in a random or structured order. In the more structured cases, many of these groups see that there are 4 sets of 16 possibilities, keeping 1 of the 4 houses constant for Ron in each set. In other words, they show the 16 possibilities where Ron is placed into Gryffindor, and all other house combinations for Harry and Hermione are attached. These groups usually get all 64 possibilities. However, many groups compile an incomplete list because of a lack of organization. In these cases, they basically use a guess-and-check strategy to find all the possibilities, which tends to be less reliable without proper organization. The second popular group approach uses a more insightful thought process, which is logically connected to a tree diagram. These groups realize that for each of Ron's 4 house choices, Harry's 4 house choices will branch off, thus giving 16 possibilities between Ron and Harry. They continue this branching process by connecting Hermione's 4 house choices to the existing 16 possibilities of Ron and Harry, concluding that there would be 64 total possibilities for the three characters.

Both approaches are representative of the multiplication principle: If there are several independent events to an outcome, you can find the number of ways the outcome can occur by multiplying together the number of ways that each event can occur. Using the multiplication principle, we can show our students an easier mathematical method for obtaining the 64 possibilities without compiling a complete list. Consider the three independent events: Ron is placed into a house, Harry is placed into a house, Hermione is placed into a house. There are 4 possible ways that each of these events can occur. Therefore, there are 64 ways that these events can occur simultaneously. This method is displayed as

$$\frac{4}{\text{Ron}}\times\frac{4}{\text{Harry}}\times\frac{4}{\text{Hermione}}=64.$$

This equation yields 64 possible ways that these characters can be placed into houses, and one of these possibilities has each of the characters being placed into Gryffindor. Therefore, there is a 1 out of 64 chance (1/64) that Ron, Harry, and Hermione will all be members of the Gryffindor house. See all 64 possible combinations in **figure 4.**

A nice extension to this probability problem is the introduction of conditional probabilities. This scenario assumes that some of the information is already known. For example, if Ron was the first character to be placed into Gryffindor, what are the chances that Harry and Hermione will also be in the Gryffindor house? Because Ron's house assignment is known, he is now limited to one house choice in the application of the multiplication principle. The equation below demonstrates this change. The students can see that, with this added condition, the total number of possible house assignments for all three characters is 16:

$$\frac{1}{\text{Ron}}\times\frac{4}{\text{Harry}}\times\frac{4}{\text{Hermione}}=16$$

Thus, the characters have a 1 out of 16 possibility (1/16) of all being placed in the Gryffindor house, assuming that Ron has already been placed there.

We find that our students are fascinated by the probabilities of this problem. They are able to extend the realization that it is lucky the characters ended up together to a very real understanding of the mathematics involved in the selection process.

When students are able to make connections between mathematics and other subjects, they appreciate the beauty of mathematics and its usefulness in the world around them. Relating mathematics to literature is an inventive way to address mathematical concepts. The exercises presented here represent just a few of the many examples we can use connecting mathematics to the Harry Potter series. We have had great success in implementing these examples in the classroom. Students are engaged by the material and see the mathematics as an extension of the storyline. While studying the mathematics of Harry's world, they are able to share in the adventures and identify with the characters. Although our examples are based on the Harry Potter books, this series is not unique. Numerous opportunities for teaching mathematics can be found in most popular literature. With a little creativity, similar mathematics problems can be applied to books that are already in a student's reading list. This interdisciplinary approach addresses the goals of a cross-curriculum view of education. The topics are presented in an interesting setting and are continually reinforced. We believe that this interconnection can only add to our students' appreciation of

G = Gryffindor H = Hufflepuff R = Ravenclaw S = Slytherin

Ron	G	G	G	G	G	G	G	G	G	G	G	G	G	G	G	G
Harry	G	G	G	G	H	H	H	H	R	R	R	R	S	S	S	S
Hermione	G	H	R	S	G	H	R	S	G	H	R	S	G	H	R	S

Ron	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H
Harry	G	G	G	G	H	H	H	H	R	R	R	R	S	S	S	S
Hermione	G	H	R	S	G	H	R	S	G	H	R	S	G	H	R	S

Ron	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R
Harry	G	G	G	G	H	H	H	H	R	R	R	R	S	S	S	S
Hermione	G	H	R	S	G	H	R	S	G	H	R	S	G	H	R	S

Ron	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S
Harry	G	G	G	G	H	H	H	H	R	R	R	R	S	S	S	S
Hermione	G	H	R	S	G	H	R	S	G	H	R	S	G	H	R	S

Fig. 4 All the possible combinations, 64, of housemates and housing are shown.

WANTED:

Writers for "Mathematics Detective"

Look around you, and you'll find mathematics everywhere: in the newspaper, at the store, in the post office, and on street signs. Some of the mathematics is accurate; some may be incorrect. But many instances can be turned into fun and fascinating activities and explorations for middle school students. Write about such an activity, including the original source of information, and send it to "Mathematics Detective," *MTMS,* NCTM, 1906 Association Drive, Reston, VA 20191-1502.

mathematics and their ability to work as problem solvers. It is an important step in the formation of their lifelong attitudes toward mathematics.

References

Demana, Franklin, and Joan Leitzel. "Establishing Fundamental Concepts through Numerical Problem Solving." In *The Ideas of Algebra, K–12,* 1988 Yearbook of the National Council of Teachers of Mathematics (NCTM), edited by Arthur F. Coxford, pp. 61–68. Reston, Va.: NCTM, 1988.

McShea, Betsy, and Maureen Yarnevich. "Connecting Mathematics to Literature: The Conversion Excursion with Harry Potter." *The Banneker Banner* (Winter 2003): 16–19.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics.* Reston, Va.: NCTM, 2000.

Rowling, J. K. *Harry Potter and the Sorcerer's Stone.* New York: Scholastic, 1997. □

Harry Potter Story Problems

Taken from http://www.mathstories.com/bookstories/Book_16_Harry_Potter.htm

Explain and Show Your Work

1) Aunt Petunia, who is 6 feet tall, came to a 4 feet fence. How much taller was she than the fence?

2) Harry needed money. He got $5 from Hagrid, $10 from Dumbledore, $0 from Dursleys and $156 from Hermione. How much did he get altogether?

3) Dudley loves bacon and can eat 5 pounds in 1 hour. How much bacon can he eat in 5 hours?

4) Harry's Nimbus 2000 flies at 60 miles per hour. How far can he fly in 5 hours?

5) Hagrid's very unusual pet, Fluffy, has 3 heads. Fluffy loves to brush his teeth. Each mouth has 69 teeth. How many teeth does he have to brush?

6) Norbert ate and ate and ate. He ate 10 pounds of food per day. After 10 days, how many pounds of food had he eaten?

7) In a big Quidditch match, Harry's teammates on the Gryffindors scored 15 points. Hufflepuff, their opponent, scored 165 points. At the last moment, Harry caught the Snitch for 150 points. What was the outcome of the match?

8) Miss Norris is on duty 17 hours a day. How many hours is she on duty over 4 days?

9) The train from London to Hogwarts travels 150 miles and takes 3 hours. What is the train's average speed?

10) Harry and Hermione are each 4 feet tall. They came upon a mountain troll that was 5 feet tall. Hermione stood on Harry's head. How much taller were they than the troll?

11) Busy Aunt Petunia spends 3 hours baking per day. How much time does she spend baking in 3 weeks?